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Can Physics Help Athletes Run Faster on a Curve Track

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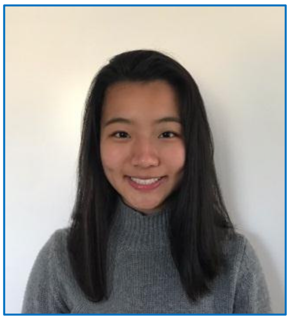
Abstract: Sprinting on a curve is slower than sprinting on a straight lane. To explain this phenomenon, various models based on a combination of biological and physical assumptions have been developed. These models depend on detailed parameters that significantly differ for each individual athlete. Here, I propose a general model solely based on kinetic theory of physics that can be universally applied to all athletes. By solving the force and torque equations for the running speed of the athletes on a curved track, I analyze sprinting speeds between the inner and outer curves. Applying the data from the classic works into my models, I find that the results and conclusions are mostly aligned with the previous works while my approach is built on the accurate physics principles and contains no uncontrollable parameters. Further I show how runners can alleviate the centrifugal effect of curved track by tilting their bodies and I quantitatively determine the optimal tilting angle for a given curvature

Key Words: Athlete, sprinting, Physics, Centrifugal effect



Katherine Han is currently an 11th grade student at Palo Alto Senior High School in California, United States. Outside of her busy school time, she would like to research and investigate physics and biology related topics. Also, she is really interested in and dedicated to exploring and researching novel physics concepts and theories. In her leisure time, she enjoys painting, playing the flute, and track and field.

Introduction

From most people's point of view, running is a sport that demands strong muscles, bigger strides and faster paces. In other words, running is more "biology" oriented. However, from a physics researcher's point of view, running is an involvement of many physics principles and techniques. For example, the push force from the foot on the ground creates another force equal in magnitude opposite in direction from the ground on the runner, causing the runner to move forward. This is based on Newton's third law. There have been many research works done on the physics and biomechanics. Most of them focus on the development of models for sprinting speed, along with the measurement and validation of some parameters in the models [1,2, 3], or studies on the effects of physiological characteristics of sprinters such as height, weight, type of build, reaction time, strength of leg muscle, etc. [4-5]. Others were done on the external conditions such as track surface, altitude, and other factors [6-7]. In this research, I will apply fundamental physics theories into running on a curved track. A practical model will be developed to illustrate the relations between the sprinting speed and the radii of curves. The goal of this work is to give runners some practical and tangible suggestions and tips when it comes to running on a curve. Before starting the detailed presentation of the research, I first make the following general assumptions:

1. Identical Conditions: In the calculations and

comparisons, I assume the same runner running the same speed with the same physical strength, i.e. the trials are completely identical and comparable. In other words, when a runner is in sprinting condition, the propel force exerted is constant.

2. Sprinting Conditions: Since the targets of this study are the effects of curve track on sprinting speed, the scenario of interest is that a runner runs through the whole curve section with sprinting speed. Assume the athletic and physical conditions are the same for the runner throughout the entire curve running. Everything before he enters and after he exits the curve is not of my interest in this research and therefore neglected.
3. Ideal Conditions: Assume the identical and perfect external conditions. The variation and effect of external conditions such as wind, track surface, altitude etc. are all neglected. Also assume that the forces generated by each foot during running are the same, and a sprinter is modeled as a rigid body.

Prior Models

One common physics model of sprinting was established on the propulsive and resistive forces acting on a human runner by Keller [1]. The sum of propulsive force F_p is constant as F_p=f*m, where f is known as the propulsive force parameter and is defined as the force per unit mass of the athlete, and m is the mass of the sprinter. The sum total of resistive force F_r acting on a runner is represented by F_r=-σmv, where v is the speed and σ is a parameter presumed constant for a given runner. Keller used a typical value of σ=0.44/sec and similar values were quoted in many other works. Keller also suggested that a well-conditioned athlete was able to sustain a maximum and nearly constant muscular effort for races of distances of 290m or less. I point out that the linear relation between the speed and the resistive force is an oversimplification that makes calculation analytically tractable. In general, more complicated forms of the resistive force could be chosen. However, it does not qualitatively affect my conclusions. Furthermore, the choice of the parameter consists with my assumption of Identical Conditions and Sprinting Conditions.

Following Igor and Philip [2], the forces acting on a sprinter at typical sprinting speed is:

$$\frac{d^2x(t)}{dt^2} = \frac{dv(t)}{dt} = f - \sigma v(t) \quad 1$$

Where x (t) is the distance measured from the start of the race and v (t) is the sprinter's instantaneous velocity, which was represented by:

$$x(t) = \left(\frac{f}{\sigma}\right)t + \left(\frac{f}{\sigma^2} - \frac{v_0}{\sigma}\right)(e^{-\sigma t} - 1) \text{ and } x(0) = 0 \quad 2$$

$$v(t) = \left(\frac{f}{\sigma}\right)(1 - e^{-\sigma t}) + v_0 e^{-\sigma t} \text{ and } v(0) = 0 \quad 3$$

The sprinter approaches the terminal (maximum) velocity V_{max}=f/σ asymptotically. While the above model is general, the computed values of the parameters f and σ differ widely from various runners as shown in Table I. It is also noticeable that the values of σ are quite different from the typical value used by Keller. Since I am interested in the comparisons between the straight and curved running, I shall focus on the same runner such that f and σ are fixed constants in comparison; this consists with the assumption of Identical Conditions.

Table I Computed values of the parameters f and σ for some athletes in source [2]

Athletes	Distance	time(sec)	*f*	σ	V_{max}(m/s)
John Carlos	100 yard	9	8.13	0.667	12.19
Bill Gaines	100 yards	9.3	13.45	1.25	10.76
Jim Hines	100 m	9.9	7.1	0.581	12.22
Tommie Smith	100 m	10.1	13.46	1.252	10.75

Igor and Philip had in fact extended their simple model to include the centripetal effects in order to predict an athlete's time for a race run on the curve. They also predicted the difference in the times for running 200m in different lanes. However, limited by the accuracy of the model itself and the parameters (f and σ) in the model, these predictions are relatively inaccurate and unreliable, especially for different runners. In my work, instead of pursuing on the prediction of speed, I am going to illustrate in general case the effects of curve track on the runners, compare the sprinting speeds in different radii curves for the same runner under the same conditions.

Greene [3] proposed models for curve sprinting performance based on the primary assumption of constant leg extension force during the run. Young-Hui and Rodger[4] explored in depth about the effects of variable ground reaction forces (GRF) in flat curve run, and performed thorough tests on the physiological characters such as leg extension force and asymmetry of the forces from both legs. While Young-Hui and Rodger's models and data could be more accurate for racing on a curve of small radii (1m to 6m) for animal locomotion, such as high-speed predator/prey chase, Greene's models and assumptions were more applicable for running on curve tracks by human beings.

Greene [3] also pointed out that lanes are unequal because of the effect of their radii on runners' speed. In order to balance centrifugal acceleration, a runner must heel over into the turn, with the approximate centreline of his body making an angle with respect to the vertical, but he did not provide a model to unveil the physics theories and the optimal conditions for tilting. Adityanarayan [7] had experiments showing that the differences of running time and running velocity are not significant among eight difference curves, but the centrifugal force of running are significantly different. It progressively increases with decrease in the length of the radius. The author further concluded that the different centrifugal forces of eight different curve radii tracks did not affect running performance of sprinters, but neither persuasive explanation nor physics theory was provided to support their observations. In this work, I will use torque theories to analyse the effects of tilting body while running on a curve. My model reaches the same conclusions as Greene and Adityanarayan; it also provides more insight on the rationale behind the observations.

Effect of curve on sprinting speed

The IAAF (International Association of Athletics Federations) Track and Field Facilities Manual 2008 stipulates dimensions for international competition by elite athletes shown in Figure 1. The Track comprises two semicircles, each with a radius of 36.50m, which are joined by two straights, each 84.39m in length. The Track has 8 or 6 lanes for international running competition. All lanes have a width of 1.22m ±0.01m.

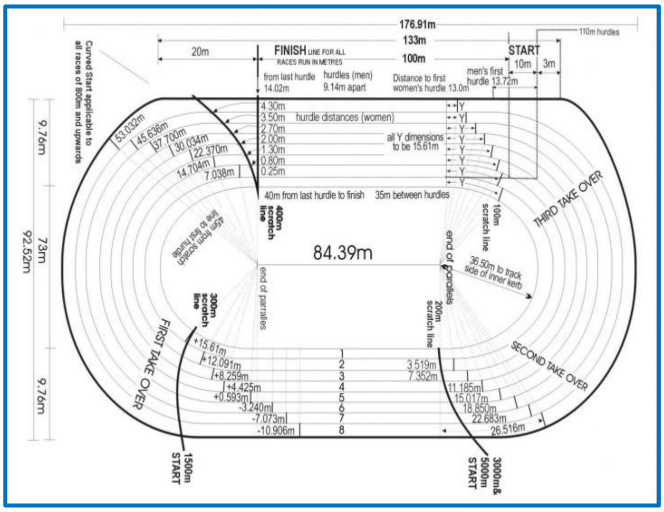


Figure 1 Dimensions of standard racing track by IAAF (source: https://www.pl-linemarking.co.uk)

In running, the runner exerts a force to the ground, yielding a reaction force from the friction known as the propel force in the horizontal direction. Figure 2 shows the free-body diagram of the forces for a runner.

Assuming the total propel force F_{propel} (same as the F_p in [2]) is a constant, which is entirely used for the forward motion in the straight lane (case A), but has to split into tangential and centripetal directions for the curved lane (case B).

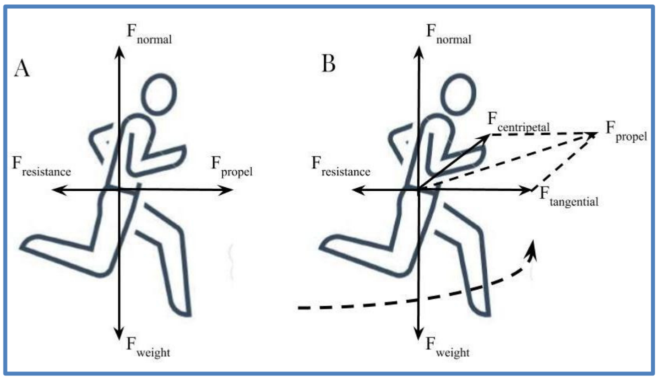


Figure 2 Free-body diagram for runner in straight and curve track

Since the centripetal force (represented as Fc) is always perpendicular to the tangential force (represented as Ft) as shown in Figure 3, we have

$$F_{propel}{}^2 = F_t{}^2 + F_c{}^2 \quad 4$$

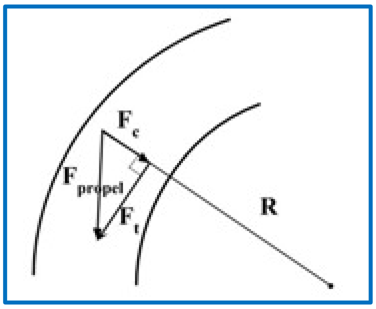


Figure 3 Analytical diagram of propulsive forces for a runner in curve track (For simplicity, the weight and the normal force are not shown in this diagram.)

Let the constant sprinting velocity of the racer on a curve track be V'_t the relationship between V'_t and F_t is the same as it between V_{max} and F_p in a straight run, i.e.

$$F_t = \sigma m V_t'$$

The relationship between F_c and V'_t is

$$F_c = m\frac{V_t'^2}{R}$$

Where R is the radius of the track curve. Given a constant F_{propel} for a human runner, the correlation between these factors can be derived as follows

F_t larger $\rightarrow$ V'_t larger $\rightarrow$ F_c larger $\rightarrow$ F_t smaller

Therefore, there must be a balance point between F_t and F_c, at which the maximum tangential speed V'_t can be achieved. By solving the equations, the model of the sprinting speed in a curve track can be obtained as:

$$V'_t{}^2 = \frac{1}{2\left(\sqrt{(R\sigma)^4+(2Rf)^2}-(R\sigma)^2\right)} \quad 5$$

or

$$V'_t{}^2 = \frac{R^2}{2\left(\sqrt{\sigma^4+\left(\frac{2f}{R}\right)^2}-\sigma^2\right)} \quad 6$$

Results and Discussions

Plugging the data from [2] (shown in Table I) into this model (Equation 5 or 6), the sprinting speeds in curve tracks for the athletes were derived as in Table II. It also shows the ratio of sprinting speed in a curve to it in a straightway.

The observations from the Table II are:

1. It proves that sprinting in a curve is slower than sprinting on a straightway. Paolo, Rodger and Alena [5] concluded in their work that non-amputee sprinters ran CCW curves 8.9% slower compared with straight running. The data of John Carlos and Jim Hines supported their conclusion. Igor and Philip [2] suggested in their work the difference between the sprinting speed in a curve track and a straight track is of the order of 0.3m/sec. My data show that the fluctuations of sprinting speed can vary dramatically among different runners, and the average is in the similar order
2. Athlete always gets higher speed on the outer Track than the inner track. However, different athletes have different levels of degradation on more curved track. Some runners keep their

speeds more stable across the lanes, which show flatter lines in Figure 4.

Table II Computed results of sprinting speed in curve tracks for the athletes

	lane 1	lane 2	lane 3	lane 4	lane 5	lane 6	lane 7	lane 8
R(m)	36.5	37.72	38.94	40.16	41.38	42.6	43.82	45.04
John Carlos								
V'_t (m/s)	11.09	11.14	11.19	11.24	11.28	11.32	11.36	11.40
V'_t/V_{max}	91.0%	91.4%	91.8%	92.2%	92.6%	92.9%	93.2%	93.5%
Bill Gaines								
V'_t (m/s)	10.49	10.50	10.52	10.53	10.54	10.55	10.57	10.58
V'_t/V_{max}	97.5%	97.6%	97.7%	97.9%	98.0%	98.1%	98.2%	98.3%
Jim Hines								
V'_t (m/s)	10.87	10.93	10.99	11.05	11.10	11.14	11.19	11.23
V'_t/V_{max}	89.0%	89.5%	89.9%	90.4%	90.8%	91.2%	91.6%	91.9%
Tommie Smith								
V'_t (m/s)	10.48	10.49	10.51	10.52	10.54	10.55	10.56	10.57
V'_t/V_{max}	97.5%	97.6%	97.8%	97.9%	98.0%	98.1%	98.2%	98.3%

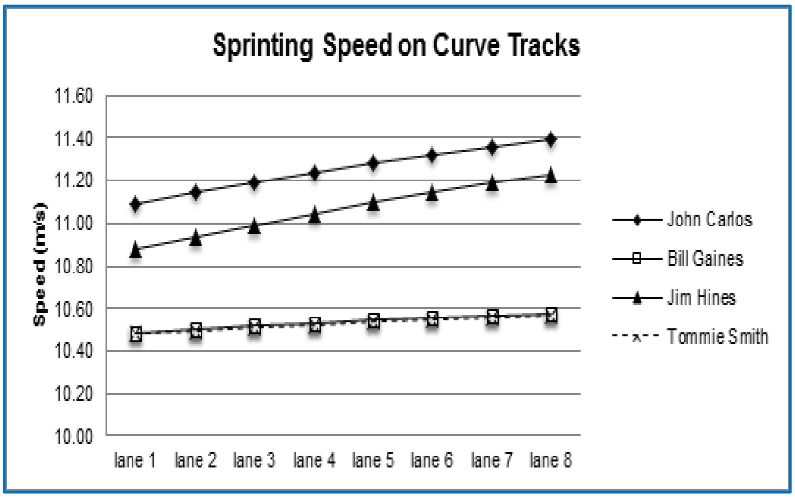


Figure 4 Velocity vs. Radius chart (V-R Chart)

Some others have dramatic variations, which show steeper lines in the figure. My result shows that Bill Gaines and Tommie Smith are better on the curve track than John Carlos and Jim Hines.

3. Advises can be drawn to the athletes for

choosing the most appropriate competition. For instance, John Carlos and Jim Hines have great speeds on straight lanes, but severe degradation on curve tracks. They should be advised to compete in straight line racing such as 100m. For races of 200m or longer, it is better for them to choose the outer track for better results. Bill Gaines and Tommie Smith were not the fastest on straight lanes, but they had better sustainability of the peak speed in curves. Therefore, they should choose races with curve track such as 200m or longer and yet better to be in the inner track.

Compare the sprinting speeds on lane 1 (the innermost lane) and lane 8 (the outermost lane) by the same runner in the same conditions. Since $F_c = m\frac{V_t'^2}{R}$, as R increases, F_c decreases. The runner on the outer track needs less centripetal force, and therefore can put more strength on tangential force F_t, and in turn gets greater V'_t, i.e. he can run faster on the outer track. These theoretical hypotheses are proven with the data in Table II which shows that a runner can always run faster on the outer lanes than the inner lanes.

Effect of Tilting

Running on a curve track, the runner will feel a force pulling him outwards from the track center. This force is known as the centrifugal force in Newtonian mechanics. It is actually an inertial force on a rotating object directed away from the axis of rotation when viewed in a rotating frame of reference. The runner has to use his physical strength to resist the “pulling away” force. This will definitely affect the sprinting speed, as well as the persistence of speed (though I assume the runner’s speed is constant all through the race). Intuitively, runners will lean their bodies to the inner direction (or say tilting) in a curve run.

According to the Physics laws, the centrifugal force on a rotating object is given as

$$F_c = m\frac{v^2}{R}$$

Where m is the mass of the object, v is the tangential speed of the rotating object, and R is the rotating radius. It is quite straightforward that the centrifugal force Fc is in a reverse proportion with the radius R, which verifies the conclusion in [7] about the centrifugal force progressively increases with decrease in the length of the radius.

The free-body diagram for a runner on a curve track is shown in Figure 5. The runner’s body is depicted as a post with the foot as the pivot and his weight and centrifugal force exerted at the center of mass (CM).

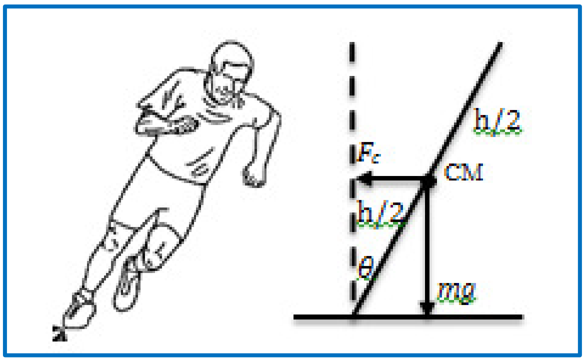


Figure 5 Free-body diagram for a runner in tilting position.

The torque by the centrifugal force is:

$$\tau_c = F_c * \frac{h}{2} * cos\theta \qquad 7$$

Where h is the height of the runner. The torque by the weight is:

$$\tau_g = mg * \frac{h}{2} * sin\theta \qquad 8$$

In the balancing (equilibrium) state, $\tau_c=\tau_g$, therefore

$$tan\theta = \frac{v^2}{gR} \qquad 9$$

Results and Conclusions

From the model of tilting angle represented by Equation 9, we can find that

- The greater the v is, the greater the tanθ is, and in turn the greater the θ is, which means the runner should tilt more;
- The greater the R is, the smaller the tanθ is and in turn the smaller the θ is which means the runner

should tilt less.

Table III Optimal tilting degree in different curve lanes for athletes

	lane 1	lane 2	lane 3	lane 4	lane 5	lane 6	lane 7	lane 8
R(m)	36.5	37.72	38.94	40.16	41.38	42.6	43.82	45.04
John Carlos								
tanθ	0.3436	0.3356	0.3280	0.3207	0.3136	0.3068	0.3002	0.2939
θ	18.96	18.55	18.16	17.78	17.41	17.06	16.71	16.38
Bill Gaines								
tanθ	0.3071	0.2981	0.2896	0.2815	0.2738	0.2666	0.2597	0.2531
θ	17.07	16.60	16.15	15.72	15.31	14.93	14.56	14.20
Jim Hines								
tanθ	0.3302	0.3231	0.3163	0.3097	0.3033	0.2971	0.2912	0.2854
θ	18.27	17.91	17.55	17.21	16.87	16.55	16.24	15.93
Tommie Smith								
tanθ	0.3067	0.2977	0.2891	0.2811	0.2734	0.2662	0.2593	0.2527
θ	17.05	16.58	16.12	15.70	15.29	14.91	14.54	14.18

When a runner sprints on a curve track, as far as he tilts his body in a proper degree so that his weight can counteract the centrifugal force, he will be able to achieve the maximum speed. This conclusion is in alignment with the observations in [7]. Table III shows the optimal tilting conditions for the athletes in [2].

Effect of the Biological Factors of the Runner

From the model above, I found that the optimal degree for tilting is irrelevant with the height and weight of the runner. That is to say, if properly tilted, runners can eliminate the physical effects on a curve track. Meanwhile, since

$$\tau_c = F_c * \frac{h}{2} * cos\theta = \frac{mv^2h}{2R} cos\theta$$

When the runner does not tilt at all, he will exert the maximum torque from the centrifugal force on the curve track.

$$\tau_{c\_max} = \frac{mv^2h}{2R} \qquad 10$$

It is obvious that if the runner is taller, faster, or weighs more, he will experience a greater torque. In other words, his running will be much heavily impacted by the curve track. Data in Table III shows that John Carlos and Jim Hines should tilt in more degrees than Bill Gaines and Tommie Smith on curve tracks. The optimal tilting angles for elite athletes are typically within 14 to 19 degrees. Even though this amount of tilting is possible for humans to achieve, it is relatively difficult for athletes to sustain such a tilt throughout the time running on the curve.

References

[1] J. B. Keller, A theory of competitive running, *Physics Today*, 26 (1973) 43.

[2] Igor Alexandrov and Philip Lucht, Physics of sprinting, *American Journal of Physics*, 49 (1981) 254-257.
[3] P. Greene, Running on Flat Turns: Experiments, Theory, and Applications, *Journal of biomechanical engineering*, 107 (1985) 96-103.
[4] Y. H. Chang and R. Kram, Limitations to maximum running speed on flat curves, *Journal of Experimental Biology*, 210 (2007) 971-982.
[5] P. Taboga, R. Kram and A. M. Grabowski, Maximum-speed curve-running biomechanics of sprinters with and without unilateral leg amputations, *Journal of Experimental Biology*, 219 (2016) 851-858.
[6] Nick Baglieri and Metzler, Why Running on the Curve of a Track is Slower, *Physics*, 111 (2016).
[7] Adityanarayan Adak, Implication of centrifugal force on curve running, *International Journal of Physical Education, Sports and Health*, 1 (2015) 14-17.

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Competing Interests

The authors declare that they have no competing interests.

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